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DISTURBANCE ABSORPTION FOR CRITICAL VARIABLES

Philip N. Jenkins
Guidance and Control Directorate
Research, Development, and Engineering Center

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This report describes the development and application of a method for designing critical variable disturbance accommodating controllers. The method is based on a state space approach. Examples are provided (complete with simulation results) to illustrate the use of the method.

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EXECUTIVE SUMMARY

Disturbance accommodating control theory is concerned with the need to meet the control objectives despite interference from the disturbances. Somtimes it is impossible to completely cancel the effect of a disturbance on all the plant's states, but it is possible to cancel the disturbance's effect on certain "critical" variables. This is known as disturbance absorption for critical variables. The original theory on this subject is described in Reference 1. The objective here was to develop an alternative method based on a state space approach. This has been done and is described herein. Example problems illustrating the application of the theory are also included, in the Appendix, with simulation results to verify the theory. The method developed here retains the drawback of earlier methods [1] in that it is not always possible to stabilize the non-critical variables. This is demonstrated in Example 1 of Appendix. This lack of stabilization is, however, easier to spot than in the older method.

TABLE OF CONTENTS

I. INTRODUCTION

Many real-world control problems require an ability to maintain adequate control in the face of disturbances. Often, this can be handled by applying the complete cancellation condition of Disturbance Accommodating Control (DAC) theory. The complete cancellation condition allows us to cancel the disturbance's effect on all the state variables. This condition can not always be met; thus, a method is needed to cancel the disturbance's effect on certain critical variables. A method for doing this is described in Reference 1. But, as it is sometimes difficult to apply, an alternate method was sought. This new method is developed and described herein.

II. ALTERNATE METHOD OF DISTURBANCE ABSORPTION FOR CRITICAL VARIABLES

In this report, we are concerned with finite-dimensional controlled linear systems governed by differential equations of the forms:

$$\dot{X} = A*X + B*U + F*W \tag{1}$$

where $X = (X_1,\ldots,X_n)$ is the state vector for the system, $U = (U_1,\ldots,U_r)$ is the control input, and $W = (W_1,\ldots,W_p)$ is the disturbance input. The matrices A, B, and F are assumed to be known and constant. The class of disturbances to be considered consists of waveform structured disturbances [2] which can be modeled by:

$$W(t) = H*Z \tag{2}$$

$$\dot{Z} = D*Z \tag{3}$$

where the vector $Z = (Z_1,\ldots,Z_\mu)$ is the state of the disturbance W. The matrices H and D are known and constant. The disturbance is not directly measurable, in general, but can be estimated using the techniques available in DAC theory described in the references.

The problem is to stabilize the state vector X to a linear subspace such that the critical state variables $X_c = (X_{c1},\ldots,X_{cm})$, $m<n$, are asymptotically stable to the null solution $X_c(t) = 0$ with the remaining (non-critical) state variables remaining bound in the subspace. Assume that the complete cancellation condition of DAC theory ($B*\Gamma + F*H \equiv 0$) is not possible for any choice of Γ. Thus, the general disturbance cancellation theory cannot be applied to the problem.

Begin by partitioning the system given in Equation 1 into critical and non-critical variables as shown below:

$$\begin{bmatrix} \dot{X}c \\ \hline \dot{X}nc \end{bmatrix} = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] \begin{bmatrix} Xc \\ \hline Xnc \end{bmatrix} + \begin{bmatrix} B_1 \\ \hline B_2 \end{bmatrix} U + \begin{bmatrix} F_1 \\ \hline F_2 \end{bmatrix} W \qquad (4)$$

Since it is desired that the critical state variables (Xc) be stabilized to zero, the control needs to be designed to make the critical state variables asymptotically stable while canceling any effects of the disturbance or non-critical states on the Xc. Thus, U could be designed as:

$$U = Up + Ud = Kp*Xc + \Gamma_1*Z + \Gamma_2*Xnc \qquad (5)$$

where $Up = Kp*Xc$ is responsible for stabilizing the critical variables to zero while $Ud = \Gamma_1*Z + \Gamma_2*Xnc$ cancels the effects of the disturbance and non-critical variables on the Xc. This requires:

$$B_1*(\Gamma_1*Z + \Gamma_2*Xnc) + A_{12}*Xnc + F_1*H*Z = 0 \qquad (6)$$

or

$$B_1*\Gamma_1 = -F_1*H \qquad (7)$$

$$B_1*\Gamma_2 = -A_{12} \qquad (8)$$

where Kp is picked to stabilize:

$$\dot{X}c = (A_{11} + B_1*Kp)Xc \qquad (9)$$

Note that the non-critical state variables are ignored, and this is unwise unless they are inherently stable, with respect to the subspace. Also, they must remain bounded in the subspace. If the non-critical variables start to grow without bound, most real system will soon run into trouble. Unfortunately, there is not much that can be done to stabilize them using the above procedure. It may sometimes be possible to exploit the non-uniqueness of the control action in stabilizing the non-critical variables. The general form for the control gain has been developed in Reference 3 and can be combined with the modified disturbance cancellation condition in Equation 7 to yield a more general form for the control parameters. The design equations for the control parameters then become:

$$B_1*\Gamma = -F_1*H \qquad (10)$$

$$K = -(C*B)^f*C*A*M*M^{\#} + [I-(C*B)^f*C*B]\phi*M^{\#} + \theta[I - M*M^{\#}] \tag{11}$$

where

- C is an m x n matrix such that $Xc = C*X$
- M is an n x n-m matrix of rank n-m spanning the subspace such that $C*M \equiv 0$
- $(!)^f$ is the Moore-Penrose generalized inverse of (!)
- $M^{\#} = (M'*M)^{-1}*M'$ where M' is the transpose of M
- ϕ is an r x n-m real parameter matrix
- θ is an r x n real parameter matrix chosen to make the critical variables asymptotically stable
- I is the identity matrix

The control action is given by $U = K*X + \Gamma*Z$ with K and Γ found from the above relations. Note that using the K given in Equation 11 automatically cancels the A12*Xnc term thus doing away with the need for Equation 8. When the generalized inverse of C*B multiplied with C*B is not equal to I, we can attempt to specify ϕ in the second term in Equation 11 to stabilize the non-critical variables.

When the above procedure is examined closely, it becomes apparent that it fails for $B1 \equiv 0$. This is due to the dependence on U to cancel the effects of the disturbance and the non-critical variables on the critical variables. If B1 is zero the control cannot directly affect the critical variables and thus the only alternative is to maneuver the non-critical state variables to cancel the disturbance effects on the Xc. Thus, it is desired that:

$$A_{12}*X_{nc} + F^1*H*Z = 0 \tag{12}$$

To accomplish this, a servo tracking system is designed where the servo command drives Xnc to cancel the disturbance effects on the critical variables. Define the servo tracking error as:

$$\varepsilon = X - Xs \tag{13}$$

with the servo command:

$$Xs = \begin{bmatrix} 0 \\ \hline \Gamma \end{bmatrix} Z \quad ===> \quad \begin{array}{l} \varepsilon c = Xc \\ \varepsilon nc = Xnc - \Gamma*Z \end{array} \tag{14}$$

then

$$\dot{\varepsilon} = \dot{X} - \dot{X}s \tag{15}$$

or

$$\dot{\varepsilon} = \left[\begin{array}{c|c} A11 & A12 \\ \hline A21 & A22 \end{array}\right] \begin{bmatrix} \varepsilon c \\ \hline \varepsilon nc \end{bmatrix} + \begin{bmatrix} 0 \\ \hline B2 \end{bmatrix} U + \begin{bmatrix} A12*\Gamma + F1*H \\ \hline A22*\Gamma + F2*H \end{bmatrix} Z - \begin{bmatrix} 0 \\ \hline \Gamma \end{bmatrix} \dot{Z} \tag{16}$$

Which when written out becomes:

$$\dot{\varepsilon}c = A_{11}*\varepsilon c + A_{12}*\varepsilon nc + A_{12}*\Gamma*Z + F_1*H*Z \tag{17}$$

$$\dot{\varepsilon}nc = A_{21}*\varepsilon c + A_{22}*\varepsilon nc + B_2*U - \Gamma*\dot{Z} + A_{22}*\Gamma*Z + F_2*H*Z \tag{18}$$

Now split the coutrol action into $U = Up + Ud$ and use Ud to cancel the disturbance terms. Cancellation of the disturbance effects on ε requires that:

$$A_{12}*\Gamma = -F_1*H \tag{19}$$

and

$$B_2*Ud = -A_{22}*\Gamma*Z - F_2*H*Z + \Gamma*\dot{Z} \tag{20}$$

Thus, choose $Ud = \Gamma_2*Z + \Gamma_3*\dot{Z}$ such that:

$$B_2*\Gamma_2 = -A_{22}*\Gamma - F_2*H \tag{21}$$

and

$$B_2*\Gamma_3 = \Gamma \tag{22}$$

where Γ must satisfy Equation 19. To obtain zero tracking error it is required that:

$$\dot{\varepsilon} = (A + B*K)\varepsilon \tag{23}$$

be asymptotically stable. Thus, design $Up = K*\varepsilon$ to place the eigenvalues of $A + B*K$ in the left half plane.

Note that if some of the rows of B_2 are also zero, then they are not directly controllable by U and another servo tracker may be needed to control them to the desired values. This is demonstrated in Example 3.

Equations 19, 21, and 22 can be combined to yield:

$$A_{12}*B_2*\Gamma_3 = -F_1*H \tag{24}$$

and

$$B_2*\Gamma_2 = -A_{22}*B_2*\Gamma_3 - F_2*H \tag{25}$$

Replace Γ in Equation 14 with $\Gamma = B2*\Gamma 3$. This eliminates the need to find Γ. Now apply the generalized inverse to obtain:

$$\Gamma 3 = -(A_{12}*B2)^f *F1*H \tag{26}$$

$$\Gamma 2 = -(B2)^f (A22*B2*\Gamma 3 + F2*H) \tag{27}$$

Equations 26 and 27, along with satisfactory stabilization of Equation 23, are sufficient for a solution. They are not necessary however, as can be easily seen from Example 3 where a satisfactory solution is obtained when no solution exists for 26 and 27.

Note that this procedure depends on the ability to maneuver the non-critical variables as desired. This may not be possible in some cases, while in others a surplus of maneuverable variables may be available giving the engineer some design freedom.

III. DESIGN PROCEDURE SUMMARY

The design procedure is summarized for the two cases developed previously. First, partition the system into critical and non-critical state variables as shown:

$$\begin{bmatrix} \dot{Xc} \\ \hline \dot{Xnc} \end{bmatrix} = \left[\begin{array}{c|c} A11 & A12 \\ \hline A21 & A22 \end{array}\right] \begin{bmatrix} Xc \\ \hline Xnc \end{bmatrix} + \begin{bmatrix} B1 \\ \hline B2 \end{bmatrix} U + \begin{bmatrix} F1 \\ \hline F2 \end{bmatrix} W$$

If $B1$ is not equal to zero, then apply the first procedure and attempt to design the control action as $U = K*X + \Gamma*Z$ where K and Γ must satisfy:

$$B1*\Gamma = -F1*H$$

and

$$K = -(C*B)^f *C*A*M*M^{\#} + [I-(C*B)^f *C*B]\phi*M^{\#} + \theta[I - M*M^{\#}]$$

where

- C is an m x n matrix such that $Xc = C*X$
- M is an n x n-m matrix of rank n-m spanning the subspace such that $C*M \equiv 0$
- $(!)^f$ is the Moore-Penrose generalized inverse of $(!)$
- $M^{\#} = (M'*M)^{-1} *M'$ where M' is the transpose of M
- ϕ is an r x n-m real parameter matrix
- θ is an r x n real parameter matrix chosen to make the critical variables asymptotically stable
- I is the identity matrix

Note, this procedure is not always successful even though K and Γ are obtained meeting the above criteria. This method depends on the non-critical state variables remaining stable and bounded in the subspace. This may occur naturally or when C*B pre-multiplied with its generalized inverse is not equal to I, we may be able to specify ϕ above such that it aids in stabilization of the non-critical variables.

For $B_1 \equiv 0$, use the second procedure. Design Γ_2, Γ_3 to satisfy:

$$\Gamma_3 = -(A_{12}*B_2)^f*F_1*H$$

$$\Gamma_2 = -(B_2)^f(A_{22}*B_2*\Gamma_3 + F_2*H)$$

and Up = K*ε to stabilize

$$\dot{\varepsilon} = (A + B*K)\varepsilon$$

The final control is then given by:

$$U = K*(X - Xs) + \Gamma_2*Z + \Gamma_3*\dot{Z}$$

where

$$Xs = \begin{bmatrix} 0 \\ \hline B_2*\Gamma_3 \end{bmatrix} Z$$

Note that meeting these conditions are sufficient for a solution, but they are not necessary. In particular, if the B_2 matrix has some zero rows, the servo tracking idea can be repeated to maneuver these indirectly controlled states and possibly meet our design criteria.

IV. CONCLUSIONS/REMARKS

A new design procedure for critical variable disturbance absorption controllers has been presented here. This procedure consists of two basic methods as described above and is demonstrated in the included examples. Simulation of the examples was done to verify the design and no problems were noted. This method retains the drawback of the earlier method (Reference 1) in that it is not always possible to stabilize the non-critical variables. This is demonstrated in Example 1. This lack of stabilization is, however, easier to spot than in the older method. The second procedure depends on the ability to maneuver the non-critical variables which is very application dependent and may or may not be possible.

It is hoped that further researchers will carry on the development and refine the method still more with hopefully the addition of necessary and sufficient conditions for a successful application of the design.

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1. Johnson, C. D., "A Discrete-Time Disturbance-Accommodating Control Theory for Digital Control of Dynamical Systems," Control and Dynamical Systems, Vol. 18, edited by C. T. Leondes, Academic Press, 1982.

2. Johnson, C. D., "Theory of Disturbance-Accommodating Controllers," Advances in Control and Dynamic Systems, Vol. 12, edited by C. T. Leondes, Academic Press, 1976, Chapter 7.

3. Johnson, C. D., "Stabilization of Linear Dynamical Systems with Respect to Arbitrary Linear Subspaces," Journal of Mathematical Analysis and Applications, Vol. 44, No. 1, October 1973, pp. 175-186.

APPENDIX

EXAMPLES

EXAMPLE #1

Objective: Design the control input (U) to stabilize the given system where X_1 is the critical variable X_c and X_2 is the noncritical variable X_{nc}.

$$\dot{X} = \begin{bmatrix} 1 & 1 \\ 0 & a \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U + \begin{bmatrix} 0 \\ 1 \end{bmatrix} W \; ; \quad X_c = [1 \;\; 0] X \qquad \text{(System)}$$

W is a piecewise constant disturbance and a is a constant. Assume:

$$W = Z \quad ==> \quad H = 1$$

and

$$\dot{Z} = \sigma(t); \qquad \text{with } \sigma(t) \text{ zero almost everywhere.}$$

Solution:

Partition the system:

$$\begin{bmatrix} \dot{X}_c \\ \dot{X}_{nc} \end{bmatrix} = \begin{bmatrix} 1 & \vdots & 1 \\ \hline 0 & \vdots & a \end{bmatrix} \begin{bmatrix} X_c \\ \hline X_{nc} \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} U + \begin{bmatrix} 0 \\ 1 \end{bmatrix} Z$$

B1 is not zero, so try the control action $U = K*X + \Gamma*Z$ with Γ and K given by equations 10 and 11 respectively.

$$B_1*\Gamma = -F_1*H \quad ==> \quad 1*\Gamma = -0*1 \quad ==> \quad \Gamma = 0.$$

$$K = -(C*B)^{\#}*C*A*M*M^{*} + [I-(C*B)^{\#}*C*B]\phi*M^{*} + \theta[I - M*M^{*}],$$

where

$$M = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad C = [1 \quad 0]$$

thus

$$M^{*} = (M'M)^{-1}M' = ([0 \;\; 1]\begin{bmatrix} 0 \\ 1 \end{bmatrix})^{-1}[0 \;\; 1] = [0 \quad 1]$$

and

$$(CB)^{\#} = ([1 \;\; 0]\begin{bmatrix} 1 \\ 1 \end{bmatrix})^{\#} = 1$$

$$K = -[1 \;\; 1]\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} + [1 - 1]\phi*M^{*} + [\theta_1 \;\; \theta_2]\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$K = [0 \;\; -1] + [\theta_1 \;\; 0] = [\theta_1 \;\; -1]$$

The control action is then:

$$U = \theta_1*X_1 - X_2$$

Substituting U into the original system yields:

$$\dot{X}_1 = X_1 + X_2 + \theta_1*X_1 - X_2 = X_1*(1 + \theta_1)$$

$$\dot{X}_2 = a*X_2 + \theta_1*X_1 - X_2 + Z$$

Now design θ_1 to make X_1 asymptotically stable. Pick $\theta_1 = -2$, then

$$U = -2*X_1 - X_2$$

and the system becomes:

$$\dot{X}_1 = - X_1$$

$$\dot{X}_2 = (a - 1)X_2 - 2*X_1 + Z$$

Note: The stability of X_2 depends on the value of a. Simulation results are shown for:

$a = -1$ ==> X_2 is stable
$a = 2$ ==> X_2 is unstable

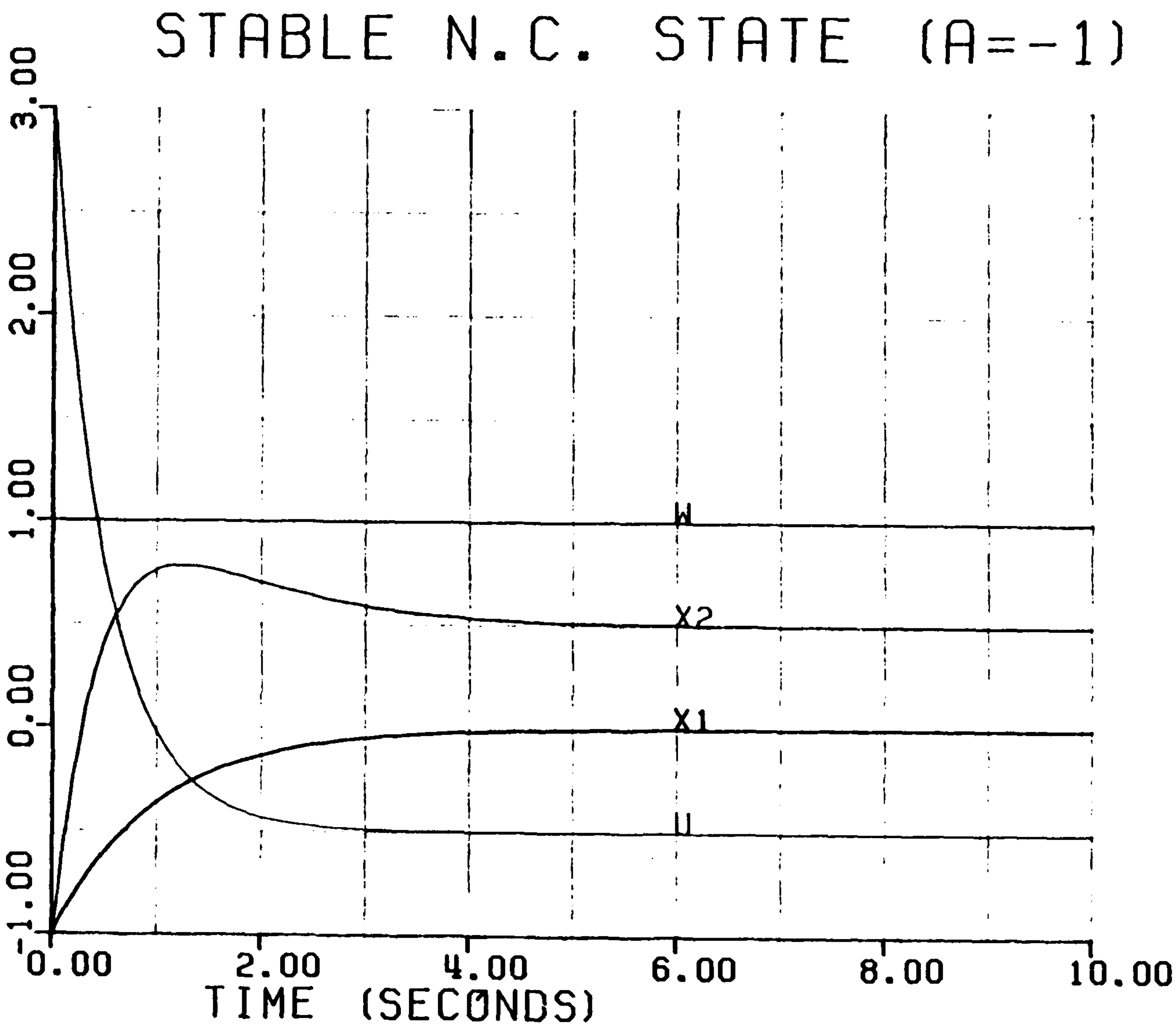

Figure A-1. Stable N. C. state (A=-1).

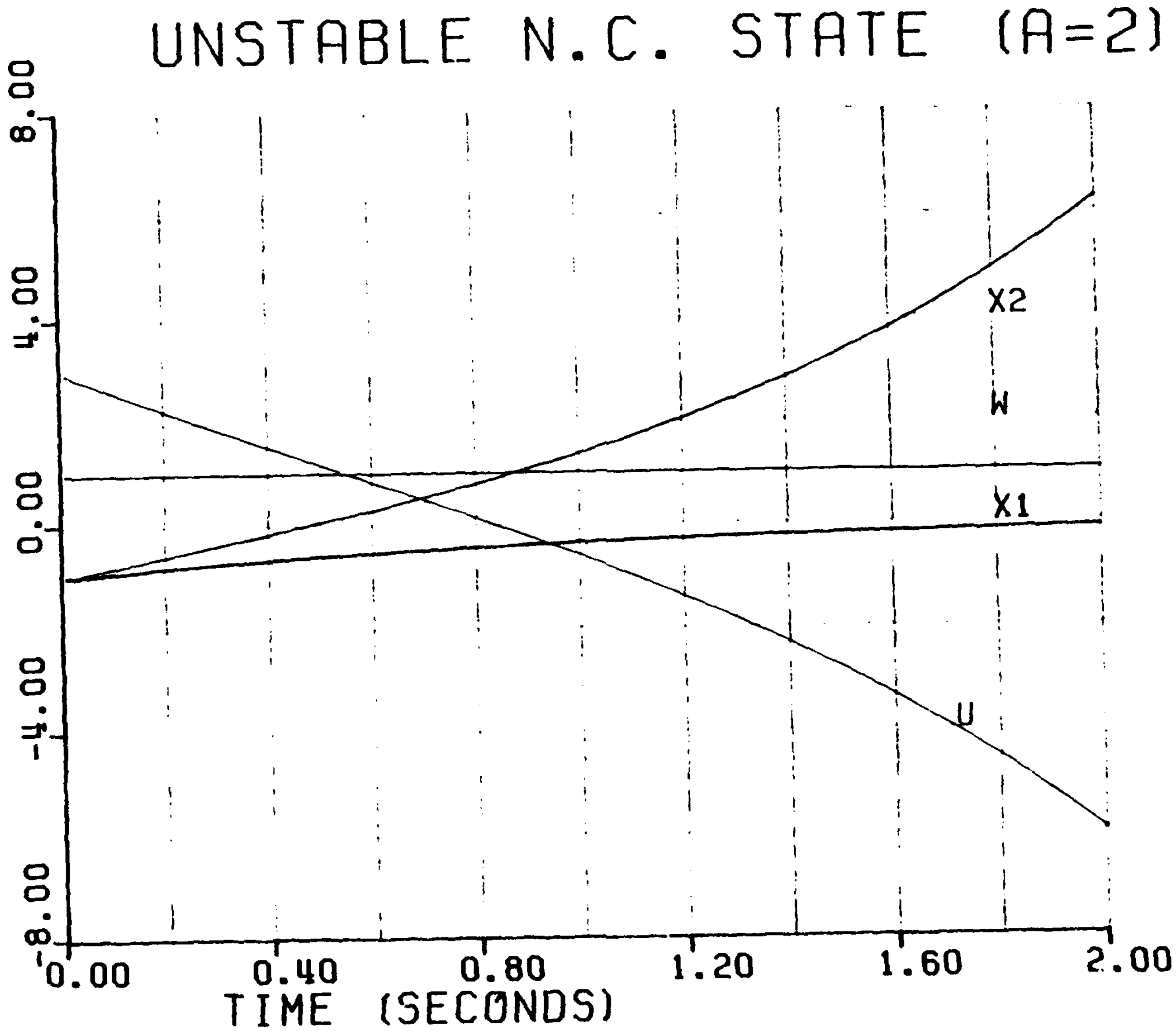

Figure A-2. Unstable N. C. state (A=2).

EXAMPLE #2

Objective: Design the control input (U) to stabilize the given system where X_1 is the critical variable X_c. X_2 and X_3 are the noncritical variables making up the vector X_{nc}.

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} U + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} W ; \qquad X_c = [1 \quad 0 \quad 0]X$$

(System)

W is a piecewise constant disturbance. Assume:

$$W = Z \implies H = 1$$

and

$$\dot{Z} = \sigma(t); \qquad \text{with } \sigma(t) \text{ zero almost everywhere.}$$

Solution:

Partition the system:

$$\begin{bmatrix} \dot{X}_c \\ \hline \dot{X}_{nc} \end{bmatrix} = \left[\begin{array}{c:cc} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ -1 & 1 & 0 \end{array}\right] \begin{bmatrix} X_c \\ \hline X_{nc} \end{bmatrix} + \begin{bmatrix} 0 \\ \hline 1 \\ 1 \end{bmatrix} U + \begin{bmatrix} 1 \\ \hline 0 \\ 0 \end{bmatrix} Z$$

$B_1 \equiv 0$ for this system, thus we use the second procedure and design

$$Ud = \Gamma_2 * Z + \Gamma_3 * \dot{Z}$$

where Γ_2 and Γ_3 are given by equations 26 and 27.

$$\Gamma_3 = -(A_{12} * B_2)^{\#} * F_1 * H = -F_1 * H = -1$$

$$\Gamma_2 = -(B_2)^{\#}(A_{22} * B_2 * \Gamma_3 + F_2 * H) = -1/2[1 \quad 1]\begin{bmatrix} -1 \\ -1 \end{bmatrix} = 1$$

Note that:

$$(B_2)^{\#} = (B_2' B_2)^{-1} B_2' = 1/2[1 \quad 1]$$

The control action U_d is:

$$U_d = Z - \dot{Z}$$

Now design $U_e = K\epsilon$ to stabilize equation 23:

$$\dot{\epsilon} = (A + B*K)\epsilon$$

Use the third order ITAE response as the model equation.

$$s^3 + 1.75 * s^2 + 2.15 * s + 1 = 0$$

This yields $K = [-1.4 \quad -0.35 \quad -1.4]$ and since

$$U_e = K * (X - X_e)$$

where

$$X_e = \begin{bmatrix} 0 \\ \text{-----} \\ B_2 * \Gamma_3 \end{bmatrix} Z = \begin{bmatrix} 0 \\ \text{----} \\ -1 \\ -1 \end{bmatrix} Z$$

The total control action is then:

$$U = U_a + U_e = -1.4X_1 - 0.35(X_2 + Z) - 1.4(X_3 + Z) + Z - \dot{Z}$$

For W piecewise constant $\dot{Z} \approx 0$ almost everywhere. Therefore:

$$U = -1.4X_1 - 0.35X_2 - 1.4X_3 - 0.75Z$$

Now design a full order estimator to estimate Z. The composite system is given by:

$$\dot{\tilde{X}} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tilde{X} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} U \quad ; \quad \tilde{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ Z \end{bmatrix}$$

and the observer with poles placed at -4 is:

$$\dot{\hat{X}} = \begin{bmatrix} -16 & 1 & 0 & 1 \\ -353 & 0 & 1 & 0 \\ -272 & 1 & 0 & 0 \\ 256 & 0 & 0 & 0 \end{bmatrix} \hat{X} + \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} U + \begin{bmatrix} 16 \\ 353 \\ 271 \\ -256 \end{bmatrix} X_1$$

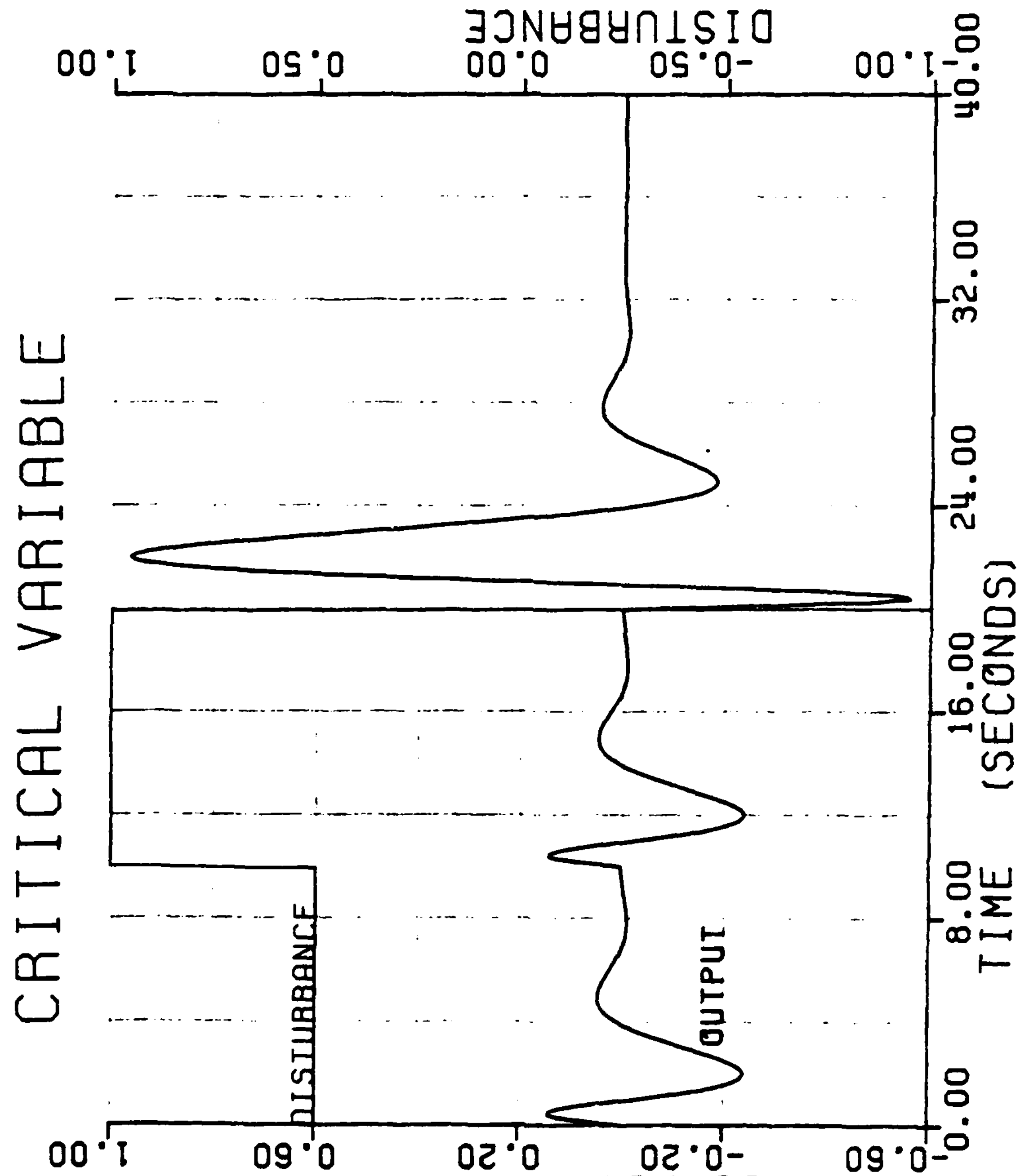

Figure A-3. Critical variable.

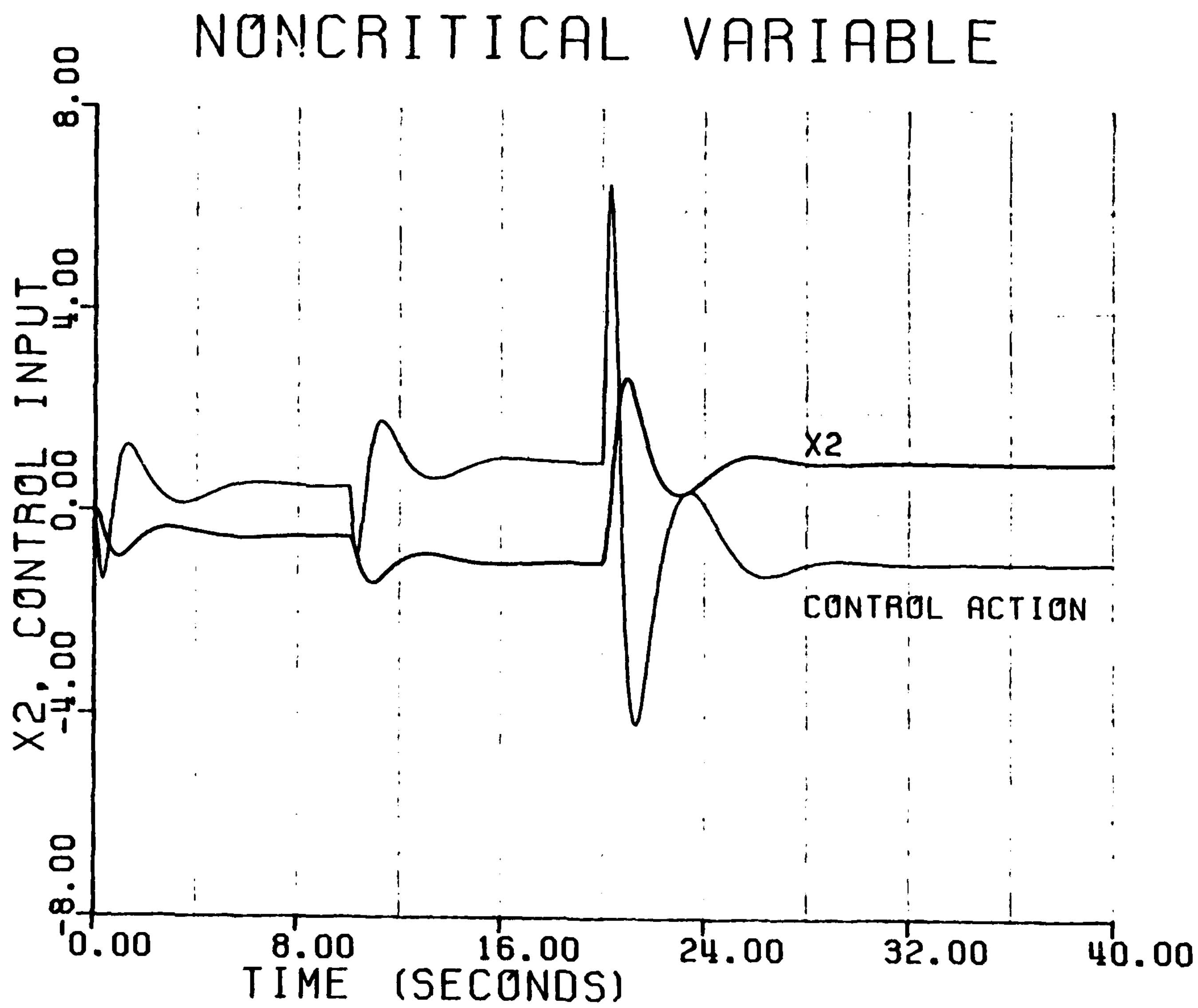

Figure A-4. Noncritical variable.

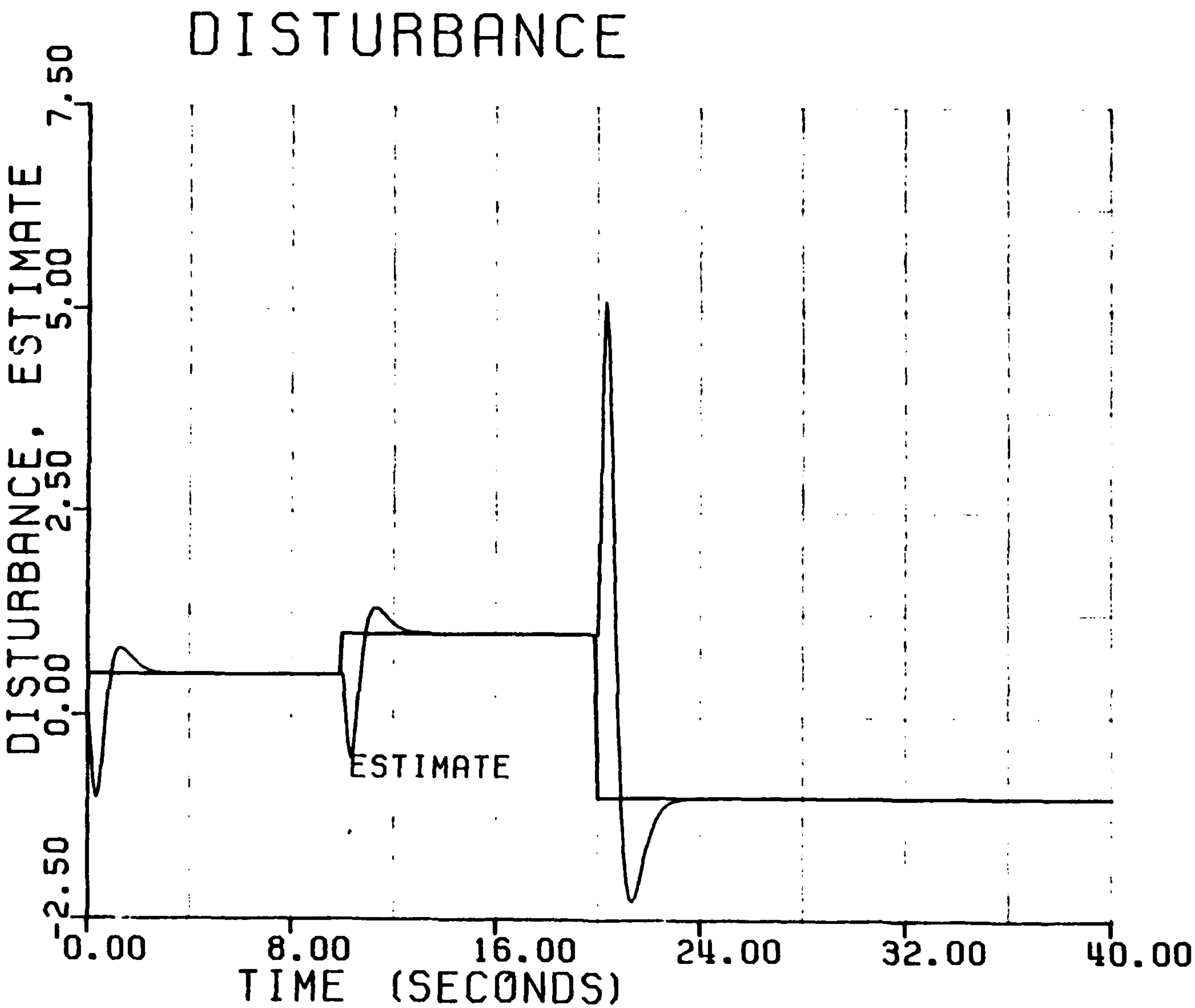

Figure A-5. Disturbance.

EXAMPLE #3

Objective: Design the control input (U) to stabilize the given system where X_1 is the critical variable X_c. X_2 and X_3 are the noncritical variables making up the vector X_{nc}.

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} W \; ; \qquad X_c = [1 \quad 0 \quad 0] X$$

W is a piecewise constant disturbance. Assume:

$$W = Z \quad ==> \quad H = 1$$

and

$$\dot{Z} = \sigma(t); \qquad \text{with } \sigma(t) \text{ zero almost everywhere.}$$

Solution:

Partition the system:

$$\begin{bmatrix} \dot{X}_c \\ --- \\ \dot{X}_{nc} \end{bmatrix} = \begin{bmatrix} 0 & \vdots & 1 & 0 \\ \hline 0 & \vdots & 0 & 1 \\ 0 & \vdots & 1 & 0 \end{bmatrix} \begin{bmatrix} X_c \\ --- \\ X_{nc} \end{bmatrix} + \begin{bmatrix} 0 \\ --- \\ 0 \\ 1 \end{bmatrix} U + \begin{bmatrix} 1 \\ --- \\ 0 \\ 0 \end{bmatrix} Z$$

$B_1 \equiv 0$ for this system, thus we try the second procedure and design

$$Ud = \Gamma_2 * Z + \Gamma_3 * \dot{Z}$$

where Γ_2 and Γ_3 are given by equations 26 and 27.

$$\Gamma_3 = -(A_{12} * B_2)^{\#} * F_1 * H$$

$$\Gamma_2 = -(B_2)^{\#}(A_{22} * B_2 * \Gamma_3 + F_2 * \dot{H})$$

Note that:

$$A_{12} * B_2 = 0$$

Inspection of these two equations reveals no solution is possible. Since B_2 has a zero row, let's try to servo control X_2 also. Use equation 19 to determine Γ and substitute Γ into the equation for the servo tracking error. Design Γ to satisfy $A_{12} * \Gamma = -F_1 * H$

$$[1 \quad 0]\Gamma = -1 \quad ==> \quad \Gamma = \begin{bmatrix} -1 \\ \tau \end{bmatrix} \; ; \quad \text{where } \tau \text{ is arbitrary}$$

Equation 16 is used for the servo tracking error. Substituting in Γ yields:

$$\dot{\epsilon} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \epsilon_c \\ \epsilon_{nc} \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} U + \begin{bmatrix} A_{12} * \Gamma + F_1 * H \\ A_{22} * \Gamma + F_2 * H \end{bmatrix} Z - \begin{bmatrix} 0 \\ \Gamma \end{bmatrix} \dot{Z}$$

$$
\epsilon = \begin{bmatrix} 0 & \vdots & 1 & 0 \\ \hline 0 & \vdots & 0 & 1 \\ 0 & \vdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_c \\ \hline \epsilon_{nc} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ \hline 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ \tau \end{bmatrix} Z + \begin{bmatrix} 0 \\ \hline 0 \\ 1 \end{bmatrix} U + \begin{bmatrix} 1 \\ \hline 0 \\ 0 \end{bmatrix} Z + \begin{bmatrix} 0 \\ \hline -1 \\ \tau \end{bmatrix} \dot{Z}
$$

$$
\dot{\epsilon} = \begin{bmatrix} 0 & \vdots & 1 & 0 \\ \hline 0 & \vdots & 0 & 1 \\ 0 & \vdots & 1 & 0 \end{bmatrix} \begin{bmatrix} \epsilon_c \\ \hline \epsilon_{nc} \end{bmatrix} + \begin{bmatrix} 0 \\ \hline 0 \\ 1 \end{bmatrix} U + \begin{bmatrix} 0 \\ \hline \tau \\ -1 \end{bmatrix} Z + \begin{bmatrix} 0 \\ \hline -1 \\ \tau \end{bmatrix} \dot{Z}
$$

The control cannot completely cancel the effect on the ϵ_{nc} states, let's try using ϵ_3 to cancel the disturbance acting on ϵ_2, in a manner similar to what was done with the critical state. Define:

$$
\mu = \epsilon - \epsilon_{sp} \quad \text{with } \epsilon_{sp} = \begin{bmatrix} 0 \\ 0 \\ \beta * Z + \alpha * \dot{Z} \end{bmatrix}
$$

Note that the set point state must now contain the derivative of the disturbance since it appears in the ϵ state equation. Taking the derivative yields:

$$
\dot{\mu} = \dot{\epsilon} - \dot{\epsilon}_{sp}
$$

which in matrix form is:

$$
\dot{\mu} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 + \beta * Z + \alpha * \dot{Z} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} U + \begin{bmatrix} 0 \\ \tau \\ -1 \end{bmatrix} Z + \begin{bmatrix} 0 \\ -1 \\ \tau \end{bmatrix} \dot{Z} - \begin{bmatrix} 0 \\ \dot{0} \\ \beta * \dot{Z} + \alpha * \ddot{Z} \end{bmatrix}
$$

or when written out becomes:

$$
\dot{\mu}_1 = \mu_2
$$

$$
\dot{\mu}_2 = \mu_3 \quad + \beta * Z + \alpha * \dot{Z} \quad + \tau * Z - \dot{Z}
$$

$$
\dot{\mu}_3 = \mu_2 \quad + U - Z + \tau * \dot{Z} - \beta * \dot{Z} \quad + \alpha * \ddot{Z}
$$

τ, β, and α must be chosen to remove the disturbance on $\epsilon 2$ which is equal to $\mu 2$. Thus letting $\beta = \tau = 0$ removes the disturbance and setting $\alpha = 1$ removes the derivative of the disturbance. U_d can be used to remove the remaining disturbance terms (those in the equation for $\mu 3$). Noting that $\beta = \tau = 0$ and $\alpha = 1$, makes it matter of inspection to determine U_d as:

$$
U_d = Z - \ddot{Z}
$$

Now we have removed the disturbance from the system and desire to stabilize it using U_s. Let:

$$
U_s = K\mu
$$

and determine K to stabilize:

$$
\dot{\mu} = (A + B*K)\mu
$$

Use the third order ITAE response as the model equation.

$$£^3 + 1.75*£^2 + 2.15*£ + 1 = 0$$

Then

$$U_p = -\mu_1 - 3.15*\mu_2 - 1.75*\mu_3$$

But $\quad \mu = \epsilon - \epsilon_{ss},$ therefore:

$$\mu_1 = \epsilon_1$$
$$\mu_2 = \epsilon_2$$
$$\mu_3 = \epsilon_3 - \beta*Z - \alpha*\dot{Z} = \epsilon_3 - \dot{Z}$$

So

$$U_p = -\epsilon_1 - 3.15*\epsilon_2 - 1.75*\epsilon_3 + 1.75*\dot{Z}$$

But $\quad \epsilon = X - X_{ss},$ therfore:

$$\epsilon_1 = X_1$$
$$\epsilon_2 = X_2 + Z$$
$$\epsilon_3 = X_3$$

which yields;

$$U_p = -X_1 - 3.15*X_2 - 3.15*Z - 1.75*X_3 + 1.75*\dot{Z}$$

The total control action is then:

$$U = U_p + U_d = -X_1 - 3.15X_2 - 1.75X_3 - 2.15*Z + 1.75*\dot{Z} - \ddot{Z}$$

For W piecewise constant $\dot{Z} \approx 0$ and $\ddot{Z} \approx 0$ almost everywhere. Therefore:

$$U = -X_1 - 3.15X_2 - 1.75X_3 - 2.15Z$$

Now design a full order estimator to estimate Z. The composite system is given by:

$$\dot{\tilde{X}} = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tilde{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} U \quad ; \quad \tilde{X} = \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ Z \end{bmatrix}$$

and the observer with poles placed at -4 is:

$$\dot{\hat{X}} = \begin{bmatrix} -16 & 1 & 0 & 1 \\ -353 & 0 & 1 & 0 \\ -272 & 1 & 0 & 0 \\ 256 & 0 & 0 & 0 \end{bmatrix} \hat{X} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} U + \begin{bmatrix} 16 \\ 353 \\ 272 \\ -256 \end{bmatrix} X_1$$

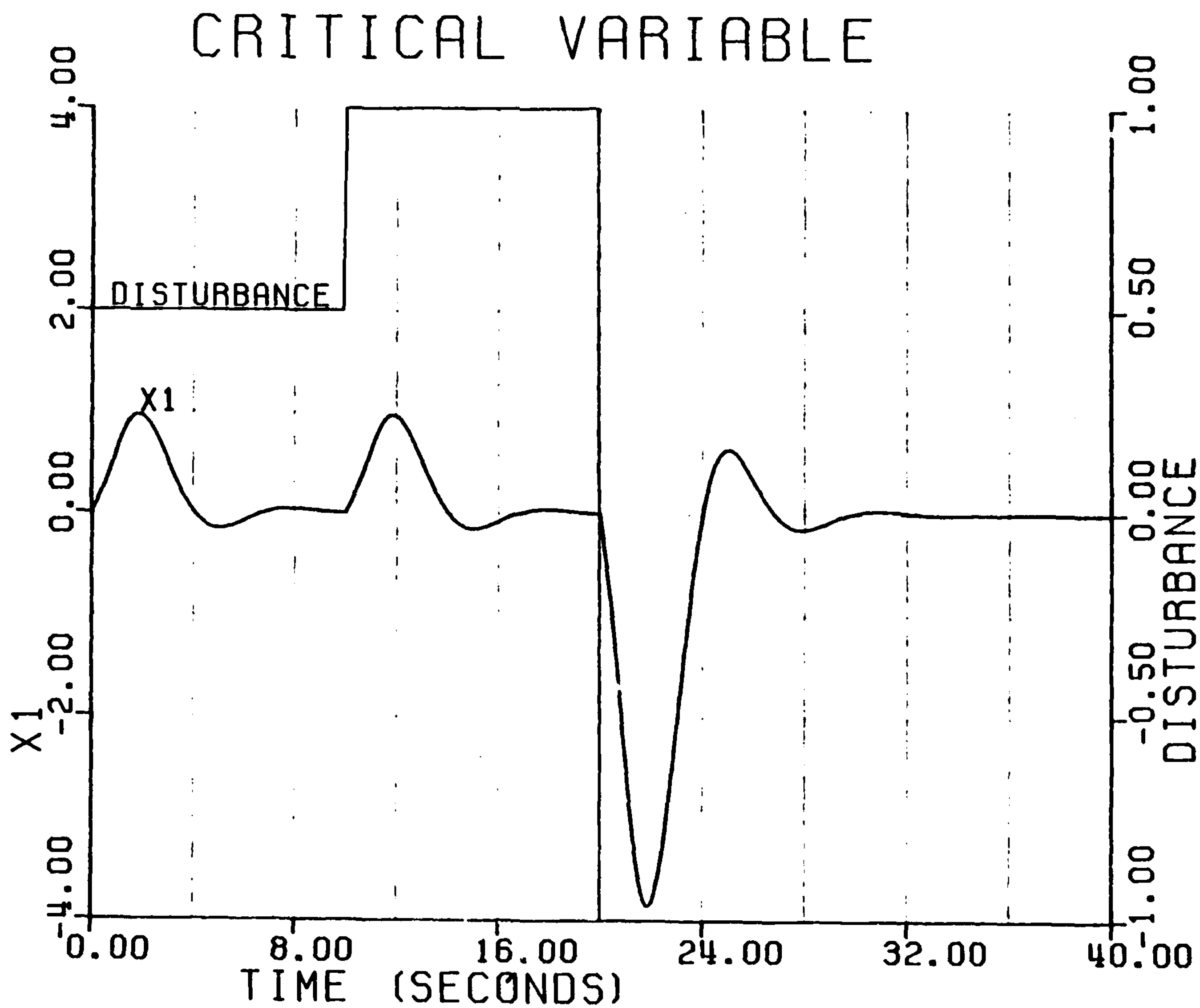

Figure A-6. Critical variable.

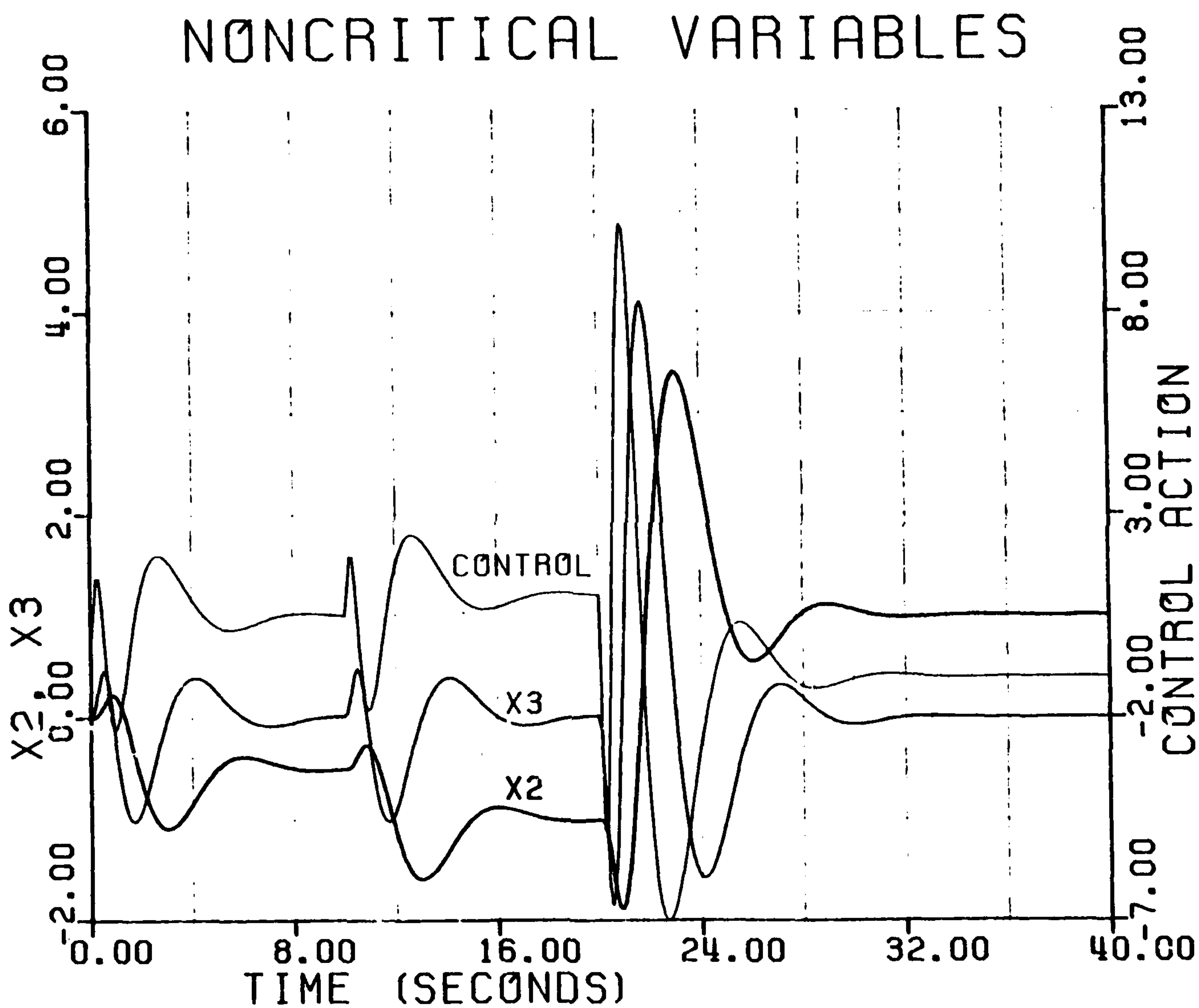

Figure A-7. Noncritical variables.

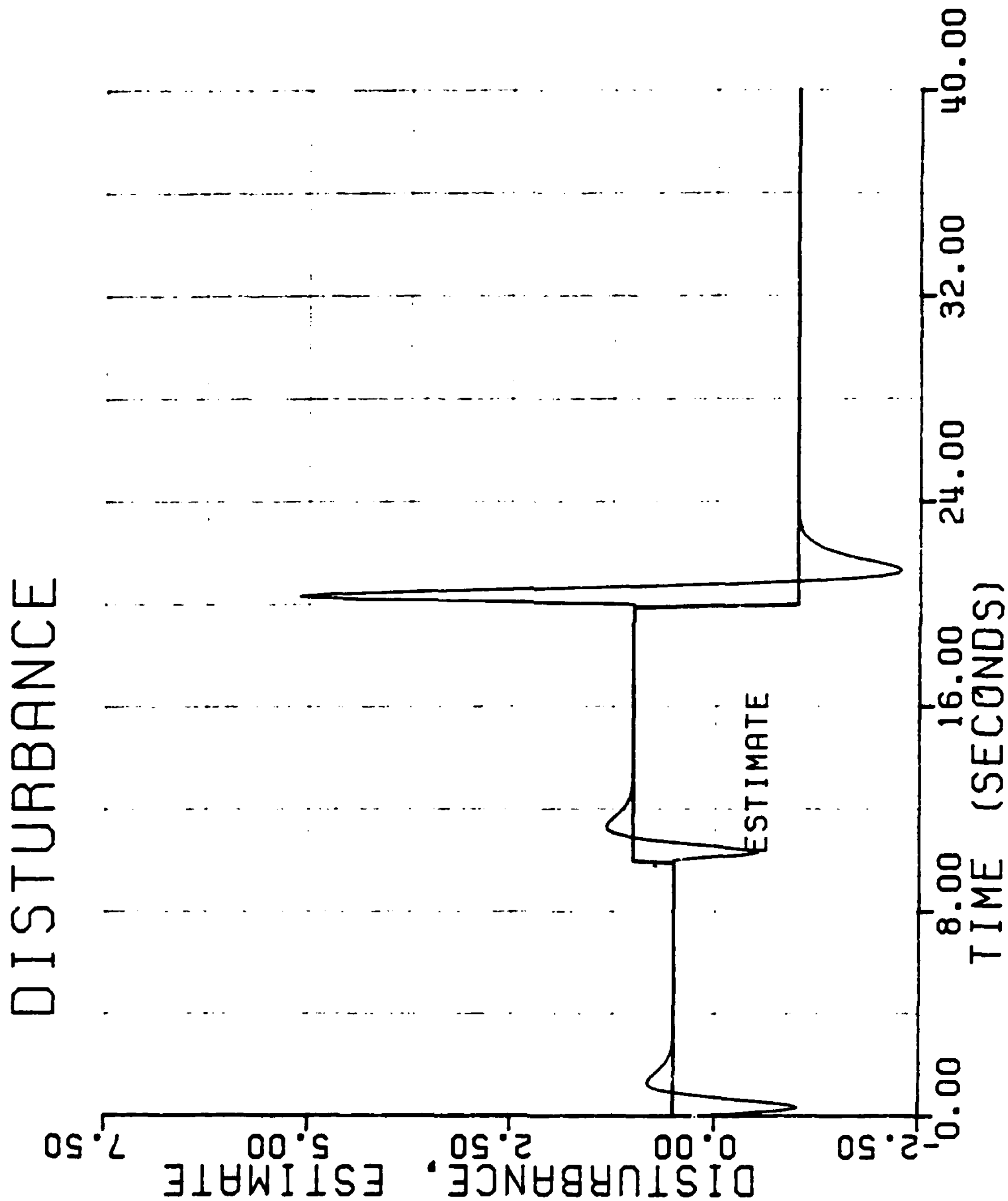

Figure A-8. Disturbance.

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